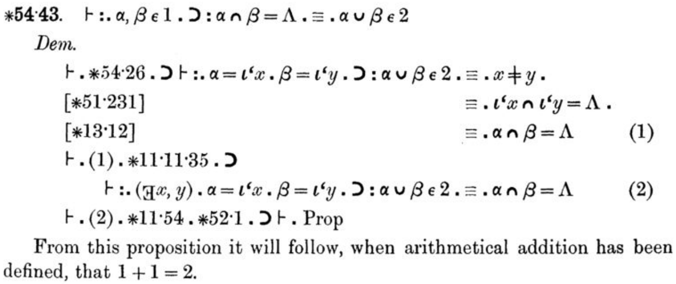


$*54{\cdot}43.\quad \vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54{\cdot}26 . \supset \vdash :. \alpha = \iota‘x . \beta = \iota‘y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

$[*51{\cdot}231] \qquad \equiv . \iota‘x \cap \iota‘y = \Lambda .$

$[*13{\cdot}12] \qquad \equiv . \alpha \cap \beta = \Lambda \qquad (1)$

$\vdash . (1) . *11{\cdot}11{\cdot}35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota‘x . \beta = \iota‘y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \qquad (2)$

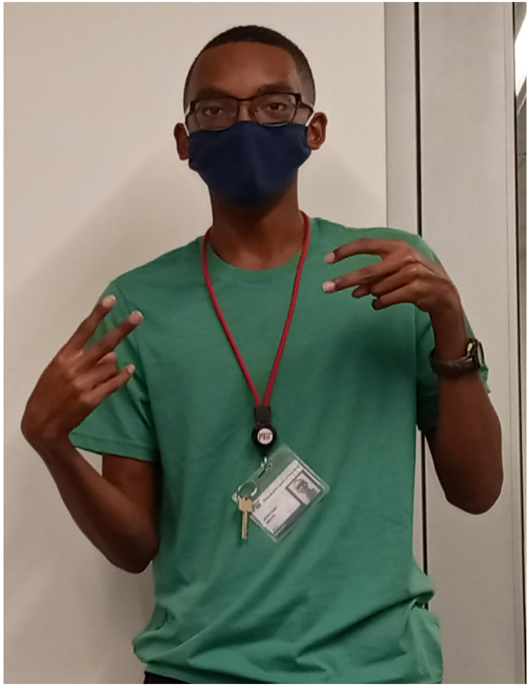
$\vdash . (2) . *11{\cdot}54 . *52{\cdot}1 . \supset \vdash .$ Prop

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

15058: Intro to Proofs

$$p \Rightarrow q$$

A Bit about Myself



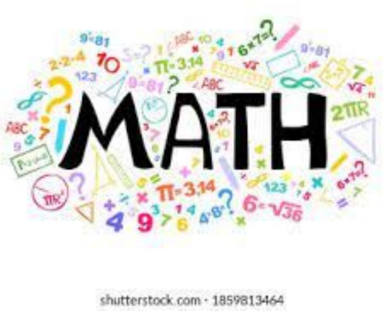
Jonathan Whyte (refer to me by first name):

- Rising sophomore at MIT
- Kinda ugly but decent at proofs
- Major: Computer Science, Data Science, and Economics (6-14)
- Hobbies: Tennis, Rubik's Cubes
- Fun fact: I have a twin sister who also goes to MIT.

Introduce Yourselves

What are Proofs?

- Wikipedia: an inferential argument for a mathematical statement, showing that the stated assumptions logically guarantee the conclusion.
- The reason that math is better than science



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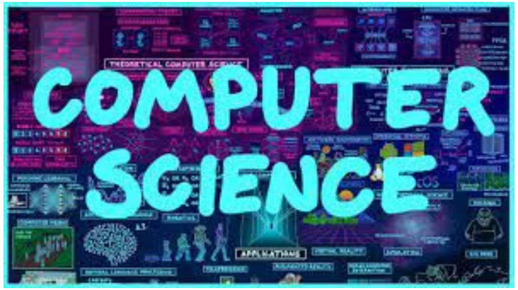
Applications

- Math, duh.
- Theoretical Computer Science
- Graduate-Level Economics
- Anything that uses theorems from Math – Science included



Goals of This Class

By the end of this class you should be able to:

- Understand various general strategies for attempting proofs and have a sound understand of why they work
- Apply various proof methods to proofs of simple theorems
- Identify logical flaws in other proofs

Few Things to Keep In Mind

- Practice makes perfect.
- Combine proof techniques.
- Proofs are Hard.
- Well-reasoned verbal argument = proof.
- Write out work.
- Feedback form.

Tentative Class Schedule

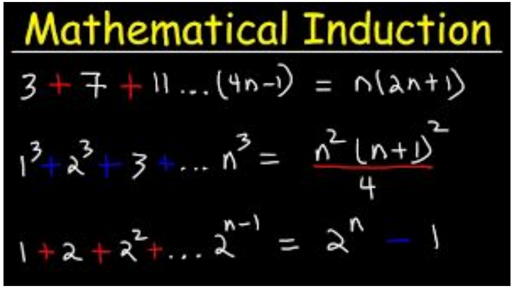
Week 1	Implications: Proving them Directly and Indirectly
Week 2	Proof by Cases, Proof by Counter-Example, Proofs of Existence (Constructive vs Non-constructive Proofs), and Proofs of Uniqueness
Week 3	Induction: What an Absolute Unit!!!
Week 4	Choose Your Adventure
Week 5	Choose Your Adventure
Week 6	Choose Your Adventure

Choose Your Adventure

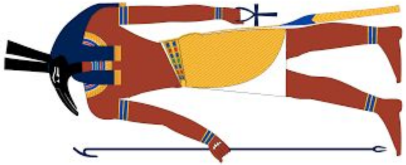
Boolean Logic

Practice Session/Gameshow

Well Ordering/Strong Induction

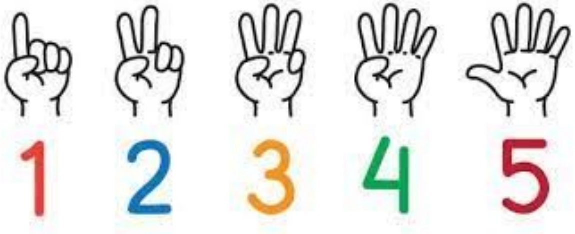


Set Theory



Predicate Logic

Combinatorial Proof

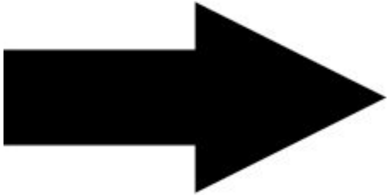


Let’s Get Rolling!

Implication

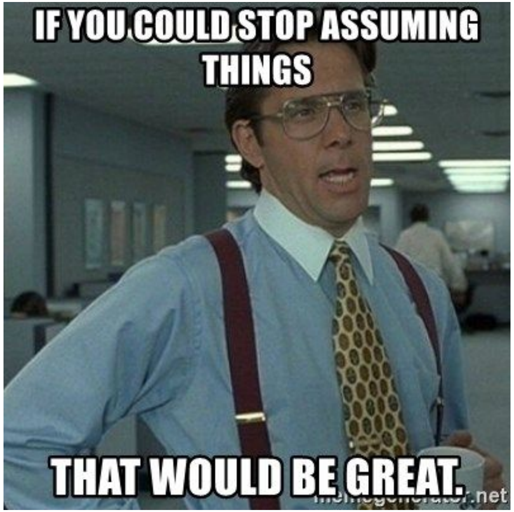
- If A, then B.
- B if A.
- If it rains, then I will not go biking.
- I will not go biking if it rains.

- $A \to B$



How to Prove Implications

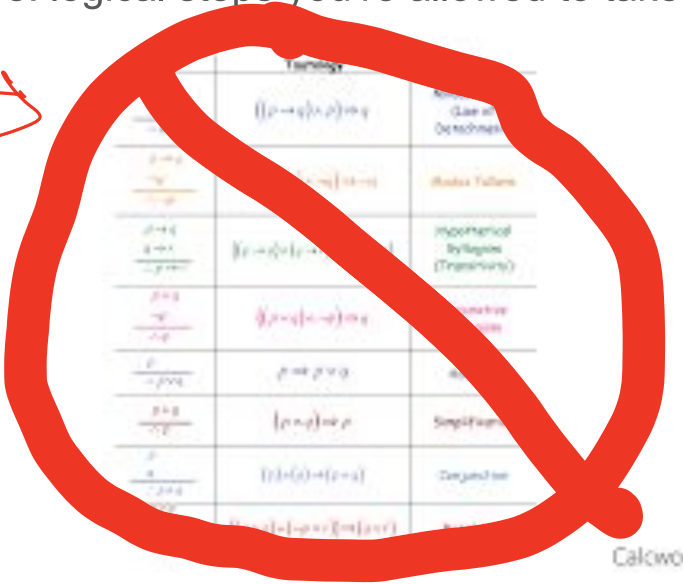
- Proof. Assume “if”, show “then.”



Direct Proofs

- Prove something is true.
- Rules of Inference: logical steps you're allowed to take

Don't Use →



Calcworkshop.com

Just Use Logic

$a + 5 = 7$ if $a = 2$

Explicitly state assumption & what we want to prove.

Proof. Assume $a = 2$. Show $a + 5 = 7$.

$a = 2$	(Assumption)
$a + 5 = 2 + 5$	(Add 5 to both sides)
$a + 5 = 7$	$(2 + 5 = 7)$

$\boxtimes$ ⇒ delimiter; says you're ending proof

Let's say the following:

- (1) "It is not sunny this afternoon and it is colder than yesterday"
- (2) If "We will go swimming, then it is sunny"
- (3) "If we do not go swimming, then we will take a canoe trip"
- (4) "If we take a canoe trip, then we will be home by sunset"

(1)–(4): Assumptions

- (5) Prove that we will be home by sunset → what we want to prove

Proof. Assume (1)-(4). Show (5).

Since it is not sunny (1), then we won't go swimming, since if we will then it would be

Sunny by (2). By (3), We don't go swimming so we take a canoe ride home. By (4), we take a canoe trip so we will be home by Sunset. ☒

If and Only If Statement

A iff B = A → B and B → A

- A if and only if B
- A iff B
- A = B
- If A then B, and if B then A.
- A ⟷ B

Proving Iffs

- Prove A iff B by proving if A then B and if B then A.

Since A happens exactly when B happens,
if A happens then B happens and vice versa.

a + 5 = 7 iff a = 2

Proof. Show $a+5=7 \to a=2$ and

$a=2 \to a+5=7$.

$a+5=7 \to a=2$ $(\to)$:

Assume $a+5=7$. Show $a=2$

$a+5=7$ (Assumption)

$a+5-5=7-5$ (Subtract 5 from both sides)

$a = 2 \quad (5-5=0;\ 7-5) = 2$

$a = 2 \Rightarrow a+5 = 7 \quad (\Leftarrow)$:

Refer to previous proof.

$\boxtimes$

Indirect Proof

- Prove something cannot be false.

Contrapositive

- Statement: If A, then B.
- Contrapositive: If not B, then not A. ⇒ negate and flip
- Contrapositive = Statement

Proof by Contraposition / Contrapositive

- State the contrapositive, then prove it.

For all integers n, n^2 is even, n is even.

Proof. Prove the contrapositive: if n is not even, then n^2 is not even. $\iff$ if n is odd, then n^2 is odd.

Assume n is odd. Show n^2 is odd.

n is odd $\rightarrow$ $n = 2m+1$ for some integer m

$n^2 = (2m+1)^2 = 4m^2+4m+1 = 2(2m^2+2m)+1$

Since $2m^2+2m$ is an integer, $2(2m^2+2m)$ is even and 1 more than that is odd. n^2 is odd. ☒

Proof by Contradiction

- Assume a statement you want to show is false. Make logical deductions to reach a clearly false statement.

For all integers n, if n^2 is even, n is even.

Proof: Assume n^2 is even. Show n is even.

Assume n is odd for contradiction.

n is odd $\rightarrow n = 2m+1$ for some integer m,

$n^2 = 2(2m^2+2m)+1 \rightarrow n^2$ is odd. But

we assumed n^2 is even. That creates a

contradiction. n must be even. $\boxtimes$

Proving Irrationality

Prove $\sqrt{2}$ is irrational.

Proof. Assume $\sqrt{2}$ is rational for contradiction.

rational $\Rightarrow \sqrt{2} = \frac{a}{b}$. Let g be greatest common factor of a and b. Let $m = \frac{a}{g}$ and $n = \frac{b}{g}$

m and n are integers, and $\frac{m}{n} = \frac{a/g}{b/g} = \frac{a}{b} = \sqrt{2}$.

m and n have greatest common factor of 1.

$$\sqrt{2} = \frac{m}{n}$$

$$2 = \frac{m^2}{n^2}$$

Proved before

$m^2 = 2n^2 \rightarrow m^2$ is even $\rightarrow m$ is even $\rightarrow m = 2i$ for some int i

$(2i)^2 = 2n^2$

$4i^2 = 2n^2$

$n^2 = 2i^2 \rightarrow n^2$ is even $\rightarrow n$ is even

2 divides m and 2 divides n, but we asserted that m and n have greatest common factors of 1. This is a contradiction. $\sqrt{2}$ must be irrational.

$\boxtimes$

Tips for Collaborative Problem-Solving

- Problem-Solving (If you're stuck):
  - Try all ideas on paper before discarding them.
  - Play around with numbers.
  - Draw pictures.
  - Try moving on to a different question and coming back to it.
- Collaboration:
  - Work together.
  - Make sure everybody understands a problem before moving on to the next one.
  - Be respectful of everybody's ideas.

Groups

Group 1	Group 2	Group 3
Marjorie Romero	Rose Dania Ferjuste	Kevin Yang
Taruna Choudhary	Chloe Park	Ashley Jisue Hong
Ursav Shah	Sabrina Berkat	Suman Samanta
Riddhi Date	Anna Sherman	Galen Xuan
	Norine Bao	